Model order reduction for the transient vibro-acoustic simulation of acoustic guitars

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Abstract
Element-based modeling techniques such as the finite element method can be used to study the characteristics of a guitar design. While these element-based formulations are versatile enough to accurately describe the complex physical behavior of acoustic guitars, they are also computationally very demanding, especially for transient analysis where the system behavior has to be computed at a very large number of points in time. This paper proposes stability-preserving model order reduction techniques to facilitate the use of finite element models for the transient simulation of acoustic guitars. In this paper a Krylov subspace technique for model order reduction is paired with a fully coupled vibro-acoustic finite element model of a YAMAHA acoustic guitar. The resulting reduced-order model is small yet accurate and stable, and allows for auralization of the sound produced by the guitar model as well as the visualization of the full transient displacement and sound fields.

1 Introduction

The design of acoustic guitars has a long-standing tradition. In recent years computational methods have been adopted to support the development and analysis of new and improved guitar designs. Simulation technology enables the designers to study the properties of a guitar in the early development stages, and the insights gained from this numerical analysis can be used to improve the guitar design. Currently, numerical studies of guitar behavior usually calculate the response in the frequency-domain or analyze mode shapes and eigenfrequencies. While this does provide some valuable information, true virtual prototyping and testing can only be done through transient analysis of the guitar response in the time-domain, as this can be used to evaluate what the guitar will actually sound like.

Element-based modeling techniques such as the finite element and boundary element method are typically employed to study the properties of a guitar design in the frequency-domain. They are flexible enough to accurately model the challenging physical properties and phenomena that are found in acoustic guitars, such as a complex geometry, strong vibro-acoustic coupling and orthotropic material properties. While these models are able to accurately describe the physical behavior of acoustic guitars, they are also computationally very demanding. For frequency-domain analysis, where only a relatively small amount of frequency lines or modeshapes have to be computed, this computational load is often still acceptable. For transient analysis however, the system response has to be numerically integrated in time to the extent that the involved computational effort is too large and transient analysis becomes practically infeasible. It is therefore that in the current scientific literature only frequency-domain computations and numerical modal analysis are covered [1, 2].

This paper proposes the use of model order reduction techniques, applied to a finite element (FE) model of the acoustic guitar, in order to effectively enable transient time-domain simulations. Model order reduction reduces the size of the finite element model (and hence the associated computational cost) while still providing an accurate description of the system dynamics. The reduction in computational effort that is achieved in this way makes it practically feasible to perform a numerical integration in time. However, classical model order reduction methods do not preserve the stability of the coupled vibro-acoustic finite element model. While this does not pose any problems for frequency-domain analysis, preservation of stability is essential for a meaningful time-domain analysis. In order to preserve stability, the authors use a recently developed method that alters the formulation of the vibro-acoustic FE model [6]. An introduction to model order reduction and the preservation of stability for vibro-acoustic FE models is presented the next section of this paper.

In section 3 the proposed method is applied to model a YAMAHA acoustic guitar. First the model reduction method is validated in the frequency-domain. The simulation results of the reduced-order model are compared to those obtained with the full FE model and the reduced-order model is also validated using experimental measurements. Next, the transient response of the reduced-order model is validated using the full FE model. Then we demonstrate the ability of the stable reduced-order model to auralize guitar sound and to simulate the full transient sound field.

Concluding remarks and suggestions for future research are made in section 4.

2 Model order reduction for transient vibro-acoustics

In this work the dynamics of an acoustic guitar are modeled using the finite element method. This method spatially discretizes the governing equations, resulting in a large system of differential equations of second order in time. This system of equations can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t), \tag{1}$$

with $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{C}^{n\times n}$ the mass, damping and stiffness matrices respectively, $\mathbf{F} \in \mathbb{C}^n$ the forcing vector and $\mathbf{x} \in \mathbb{C}^n$ the system state, where n is the number of degrees of freedom (DOF) in the system (which is often a large number). The time-domain solution to this set of equations is unique when the initial conditions and the forcing vector are specified, but no general algorithm to find the exact solution (in a finite amount of time) exists. An approximation to that solution can be computed by numerical integration of the above equations over time. The system of equations in (1) is then discretized in the time dimension and the solution is constructed in a step-by-step approach, starting from the point in time at which the initial conditions are given. In each of these many time steps an algebraic system of size $n \times n$ has to be solved.

Because a large algebraic system of equations has to be solved many times, the construction of a time-domain solution is computationally very expensive. In order to reduce the computational effort, model order reduction techniques substitute the large system of equations in (1) by a much smaller one which still produces an accurate approximation to the time-domain solution. This can be done by projecting the system of equations onto a pair of projection matrices $\mathbf{V}$ and $\mathbf{W} \in \mathbb{C}^{n\times r}$, with $r \ll n$. The resulting small system of equations is called the reduced-order model (ROM):

$$\mathbf{M_r}\ddot{\hat{\mathbf{x}}}(t) + \mathbf{C_r}\dot{\hat{\mathbf{x}}}(t) + \mathbf{K_r}\hat{\mathbf{x}}(t) = \mathbf{F_r}(t), \tag{2}$$

with

$$\begin{aligned} \mathbf{M_r} = \mathbf{W^T M V} \qquad \mathbf{C_r} = \mathbf{W^T C V} \\ \mathbf{K_r} = \mathbf{W^T K V} \qquad \mathbf{F_r} = \mathbf{W^T F} \\ \mathbf{x} = \mathbf{V}\hat{\mathbf{x}}. \end{aligned} \tag{3}$$

When using the ROM in (2)-(3) instead of the full FE model in (1) for the time integration, only a system of size $r \times r$ has to be solved in every time step, which greatly reduces the computational effort. Effective model order reduction techniques strongly reduce the size of the system ($r \ll n$) while still maintaining good accuracy. This can be achieved by careful construction of the projection matrices $\mathbf{V}$ and $\mathbf{W}$. Various methods for doing so can be found in the scientific literature [3]. In this work the projection matrices span a Krylov subspace, and are constructed using the second-order Arnoldi (SOAR) algorithm [4, 5].

For the use of these ROMs in time-domain applications, there is an additional point of concern: the model order reduction methods should preserve the stability of the original finite element model. The mutual vibro-acoustic coupling in the guitar models makes this a non-trivial task, as it either leads to a loss of symmetry or a loss of positive definiteness in the system matrices. In [6] two possible solutions to this problem are developed. The first option consists of using a different formulation for the coupled vibro-acoustic problem, while the second method extends the projection basis to retain stability. Since the second method considerably increases the reduced order model size, we adopted the first method (modified formulation) in this paper.

The ROM that is obtained in this way is stable, so it can be used for both frequency-domain and time-domain analysis.

3 Case study

In this section we use the finite element method and stability-preserving model order reduction to simulate the behavior of a YAMAHA acoustic guitar both in the frequency-domain and the time-domain.

3.1 Model and modeling assumptions

The geometry of the studied guitar model is shown in figure 1, where half of the top plate is hidden to show the complex inner structure. The air both inside and outside the guitar is modeled, and a strong two-way coupling between the guitar structure and the air is taken into account. This means that structural vibrations of the guitar body produce acoustic waves, but in turn also acoustic waves may excite structural vibrations. Since the guitar body is relatively thin-walled and flexible while high acoustic pressures appear in the guitar sound box, it is important to take this two-way coupling into account.

The guitar is assumed to be in an open, unbounded domain. However, since the computational domain needs to be finite, an appropriate boundary condition that mimics free radiation has to be selected and applied to the outer edge of the computational domain. Multiple methods exist for this purpose ([7]) but most of them lead to frequency-dependent, nonsymmetric or indefinite system matrices, which are difficult to incorporate in the stability-preserving model order reduction method of [6]. One possibility is to impose a Sommerfeld radiation condition on this boundary, since it is equivalent to assigning an impedance boundary condition (using the characteristic impedance of the medium) and is therefore well-suited to our approach. In order that this Sommerfeld condition accurately represents free radiation, the boundary has to be placed far enough from the radiating object, which means that the computational domain can become fairly large. Since model

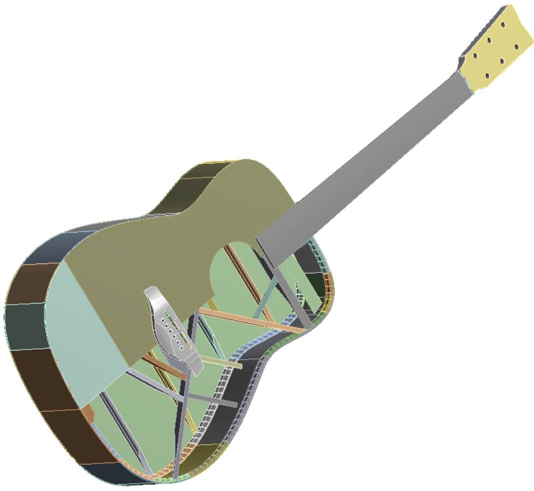

Figure 1: Geometry used in the finite element model. Half of the top plate is hidden to reveal the complex inner guitar structure.

order reduction is applied to reduce the model size anyway, this is not such a big drawback. Figure 2 shows the computational domain including the structural guitar geometry, the acoustic domain and its boundaries. It is worth mentioning that recently the stable model-order reduction framework for vibro-acoustic FE models from [6] has been extended to also handle infinite elements, which provide a better approximation of the free radiation conditions than the Sommerfeld condition (which is perfect only at an infinite distance) [8].

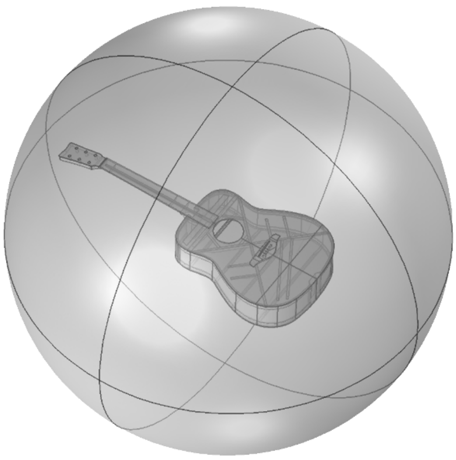

Figure 2: Guitar geometry, acoustic domain and boundary.

The strings of the guitar are not modeled. Instead, a point force is applied to the guitar bridge. This is done in order to study the guitar body design only, but in future research the model can be extended to include strings. For the time-domain simulations, the initial conditions assume that both the guitar body and the surrounding fluid are at rest.

The finite element mesh is shown in figure 3. Tetrahedral solid elements with quadratic shape functions are used to model the structural part with a spatial resolution of at least 5 elements per wavelength at well over 800 Hz. Tetrahedral elements with linear shape functions are used for the acoustics with a resolution of at least 8 elements per wavelength at well over 800 Hz. This results in a total FE model size of 564 228 degrees of freedom.

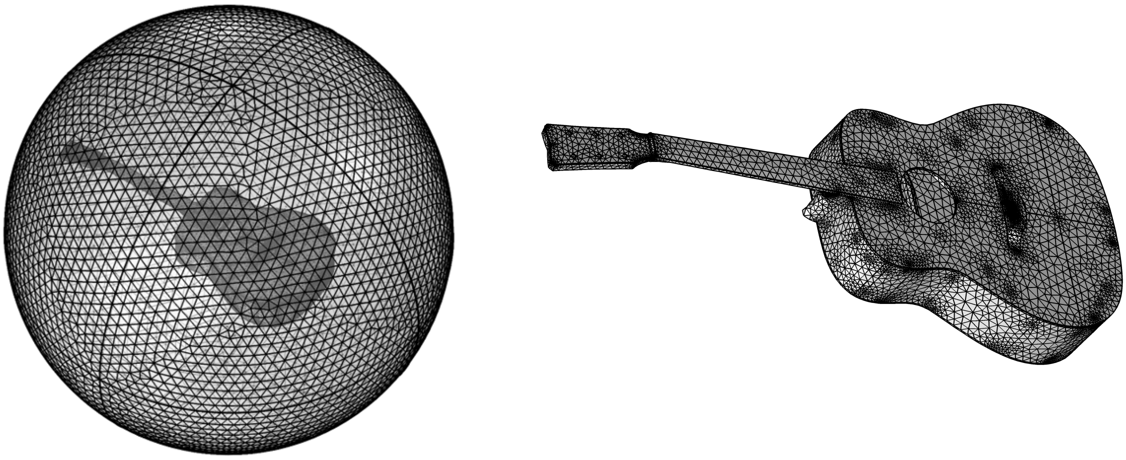

Figure 3: Finite element mesh for the guitar and surrounding air.

For the model order reduction a Krylov subspace is constructed using 13 expansion points of order 20, resulting in a reduced order model with a total size of 260 degrees of freedom. The method presented in [6] is used in order to obtain a stable ROM.

3.2 Frequency-domain analysis

With a point force acting on the guitar bridge as excitation and the response taken at a point facing the guitar sound hole, the frequency response function of acoustic pressure over force is computed using both the full FE model and the ROM. The results are shown for comparison in figure 4, where the axis labels have been replaced for reasons of confidentiality. Figure 5 displays the relative amplitude error ε of the ROM which is defined as

$$\varepsilon = \frac{|FRF_{\text{ROM}} - FRF_{\text{full FE}}|}{|FRF_{\text{full FE}}|}. \tag{4}$$

Figures 4 and 5 demonstrate that the reduced-order model approximates the original finite element model very well up to a frequency of 5x (the true frequency is not disclosed for reasons of confidentiality). This is also the frequency where the finite element mesh has a spatial resolution of 5 quadratic elements per wavelength for the structural domain and 8 linear elements per wavelength for the acoustic domain. If we consider this to be the upper frequency limit where the full finite element model accurately models the underlying partial differential equations, then it is fair to say that the ROM is an accurate approximation of the FE model over its full range of application. In fact, the expansion points and orders for the construction of the Krylov subspace were chosen such that an accurate approximation over this frequency range was obtained. The ROM was *designed* to be accurate in this range, and it took only 260 DOF to realize this. Note that using a more efficient procedure for the selection of the expansion points, such as IRKA, may result in an even smaller ROM with similar accuracy [9].

The model sizes and computational times for the model reduction step and the calculation of the FRF are summarized in table 1. The construction of the ROM does require some computational effort, mainly in computing the basis vectors for the Krylov subspace, but this a one-time overhead cost. Once the ROM has been obtained, the benefits of having a very small yet accurate model can be fully exploited. The use of an efficient iterative algorithm for the selection of the expansion points used to construct the Krylov subspace may enable even further reduction of the ROM size (and associated computational cost of the simulations),

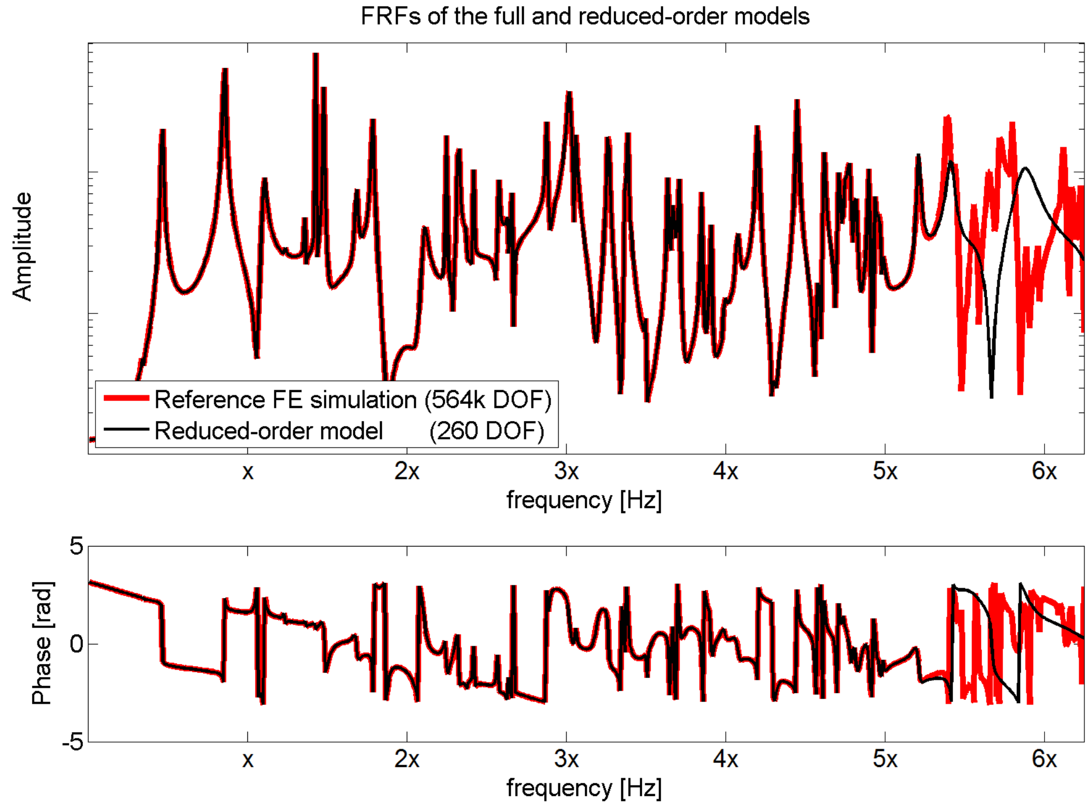

Figure 4: Frequency response functions (FRF) of acoustic pressure over force, obtained using the full finite element model and the reduced-order model. The FRFs match well up to 5x Hz.

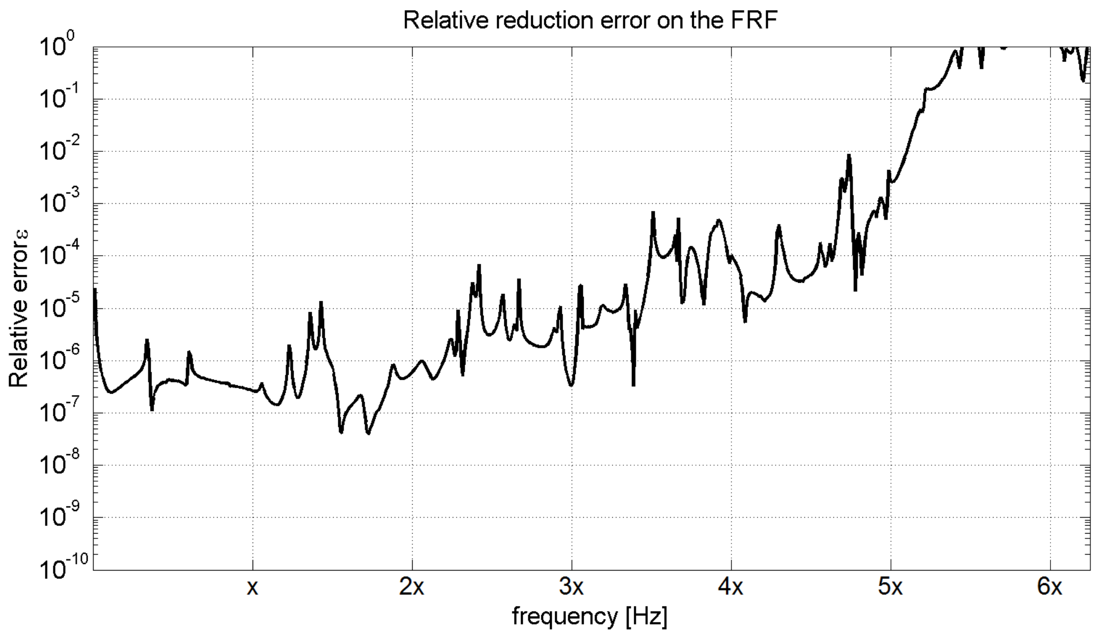

Figure 5: The relative amplitude difference between the FRFs obtained using the full finite element model and the reduced-order model, which serves as a measure for the reduction error. This relative error ε remains smaller than 1% up to 5x Hz.

but this usually comes at the expense of a higher computational (overhead) cost for the model reduction step.

	finite element model	reduced-order model
Number of DOF	564 228	260
Model reduction time	-	2,7h
FRF computation time	76,4h	3,6s

Table 1: Model size and computational times for both the finite element model and the reduced-order model. The computational times depend on the system used and are only meant to be indicative.

We have shown the ROM to be a very compact yet accurate approximation of the finite element model. In order to assess the overall quality of the ROM and to evaluate its ability to predict real guitar behavior and characteristics, an experimental validation is required. This enables us to quantify the impact of modeling errors, including but not limited to errors in the underlying equations, realization and implementation of initial and boundary conditions, deviations in geometry and material properties, discretization and reduction errors. To this end a hammer impact test was performed on a YAMAHA guitar, where the guitar bridge was excited and a microphone was used to capture the response at a distance of 15cm from the top plate. The resulting FRF is depicted in figure 6, where it is compared to the corresponding FRF computed using the ROM. It can be seen that the numerical model is able to adequately predict the experimental response, especially when considering that a guitar is a complex system with a high degree of uncertainty and variability with respect to material properties and the influence of environmental conditions (such as temperature and humidity).

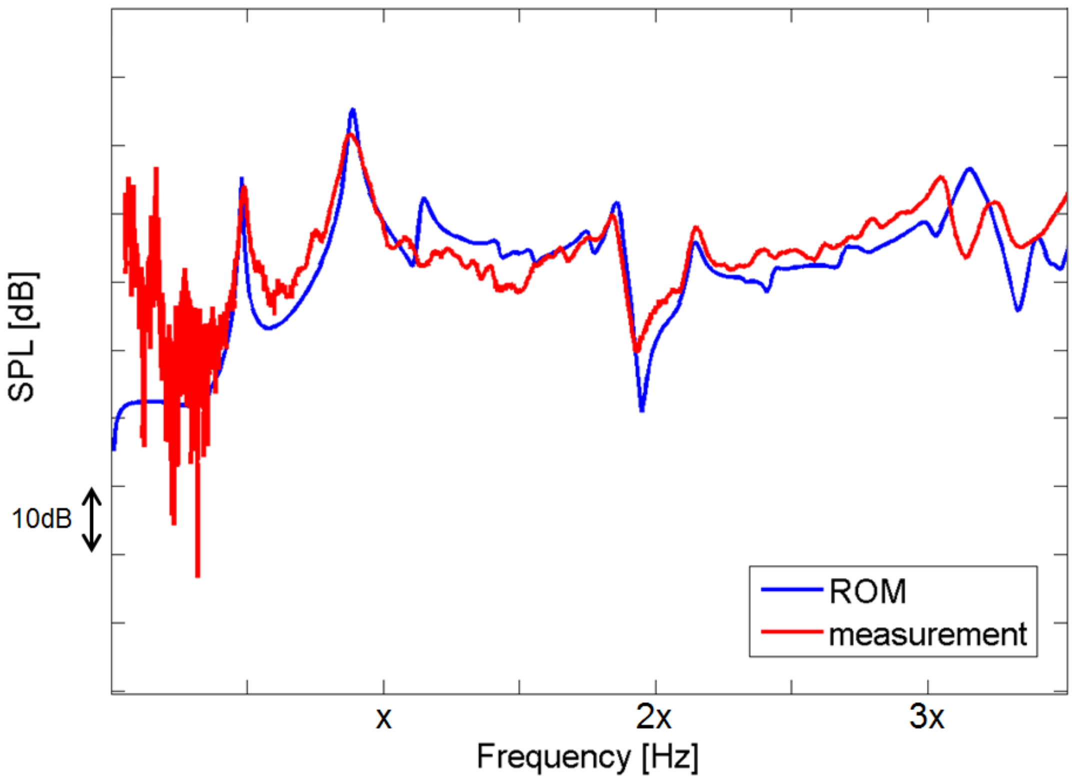

Figure 6: The ROM is able to predict the experimentally measured FRF with sufficient precision.

This confirms that the use of model order reduction techniques combined with a finite element model of an acoustic guitar allows for accurate and efficient numerical evaluations of actual guitar behavior.

3.3 Time-domain analysis

In section 3.2 we demonstrated the effectiveness of model-order reduction techniques for the simulation of guitar dynamics in the frequency-domain. For transient simulations the continuous-time system of equations in (1) or (2)-(3) has to be numerically integrated in time. Numerous methods for the time integration of dynamical systems exist. In this work we have chosen to use the generalized-α method [10] to demonstrate the performance of both the FE model and the ROM, but a different time integrator would yield similar results. The generalized-α method is an implicit time-integration method and therefore unconditionally stable, as long as the underlying model itself is stable.

Since the reduced-order models produced by the method developed in [6] retain stability, their transient dynamics can be compared to those of the finite element model. With the guitar and surrounding fluid initially at rest and the guitar bridge excited by a point force, the acoustic pressure is recorded in a field point facing the sound hole. The results from both models as well as the difference between the two simulated responses are displayed in figure 7. The simulated time interval is only short since the size of the finite element model makes time-domain calculations very uneconomical. Again the labels on the horizontal axis have been replaced in order to not disclose any confidential information on the guitar design. It can be observed that also in the time-domain, the reduced-order model provides a very accurate approximation of the full finite element model dynamics.

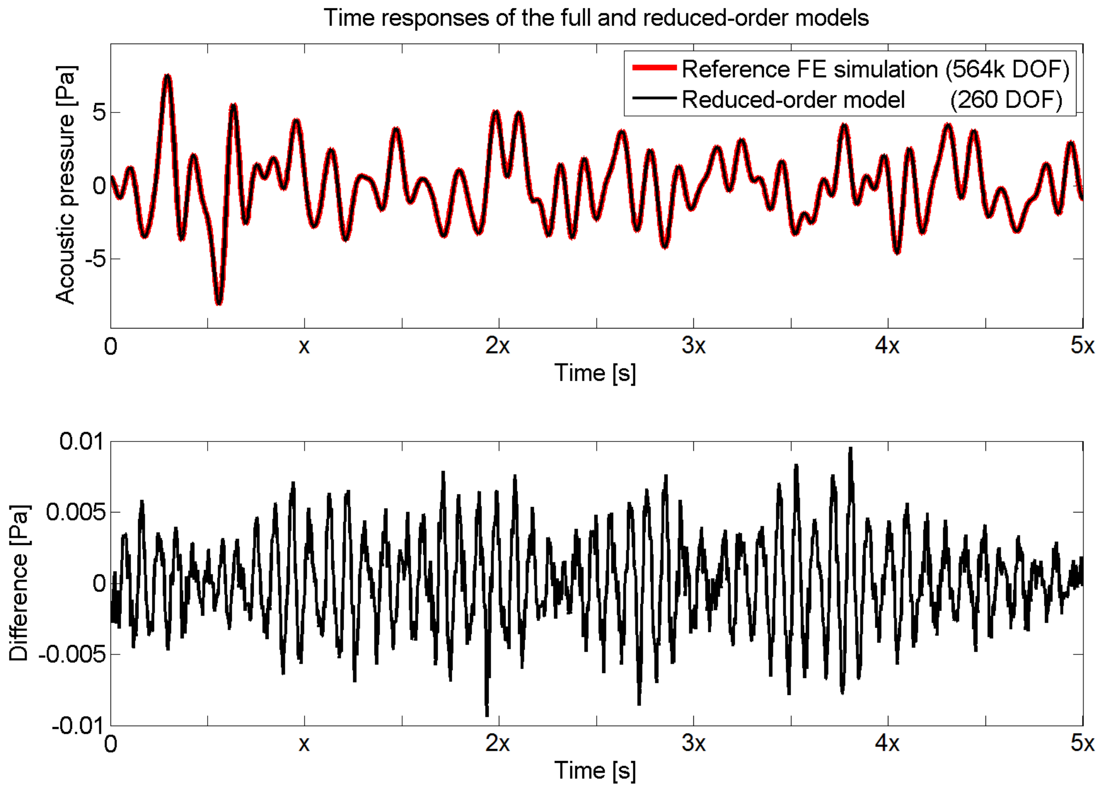

Figure 7: The time-domain response of the reduced-order model closely matches the response computed using the full finite element model.

Where it is practically infeasible to perform time-domain simulations over a meaningful time span with the finite element model because of its size, the reduced-order model does not suffer from these restrictions. As an illustration the same time-domain response of the ROM, but now simulated over a longer time span, is shown in figure 8. With the stability-preserving model order reduction techniques it has become possible to perform accurate yet fast time-domain simulations of the response of acoustic guitars. This allows guitar designers to analyze the transient displacement and sound pressure fields (as shown in figure 9), and also to auralize the sound of a virtual guitar design.

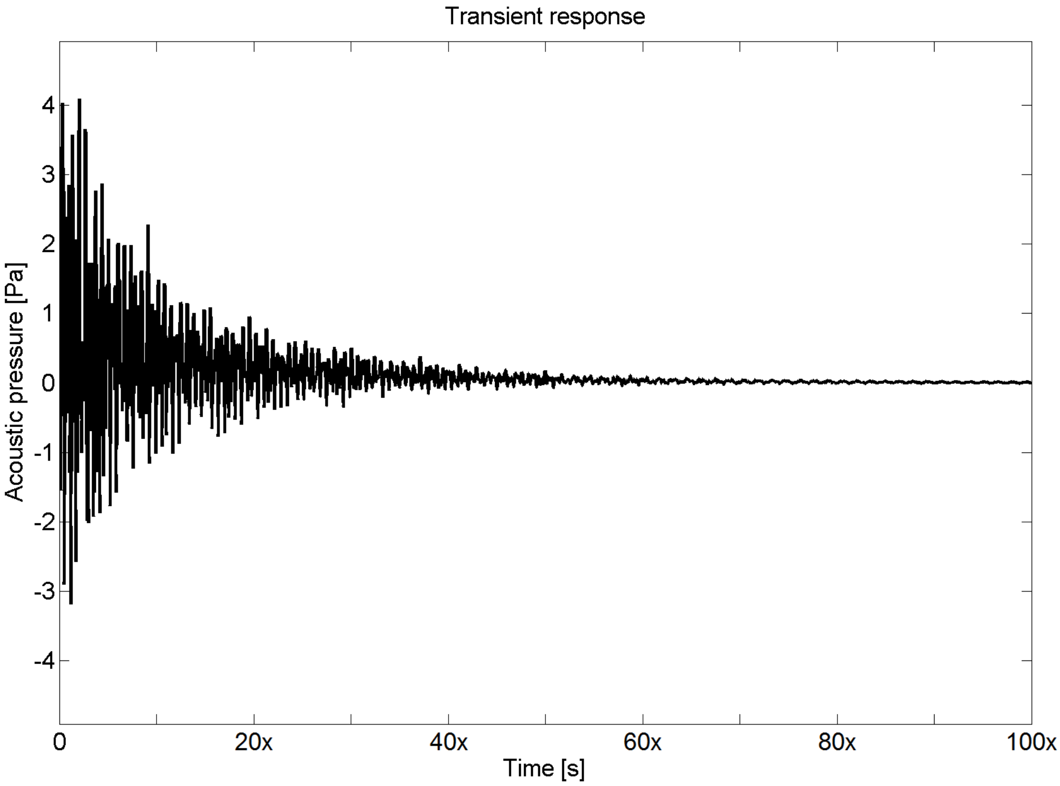

Figure 8: The stable reduced-order model allows for efficient time-domain simulations.

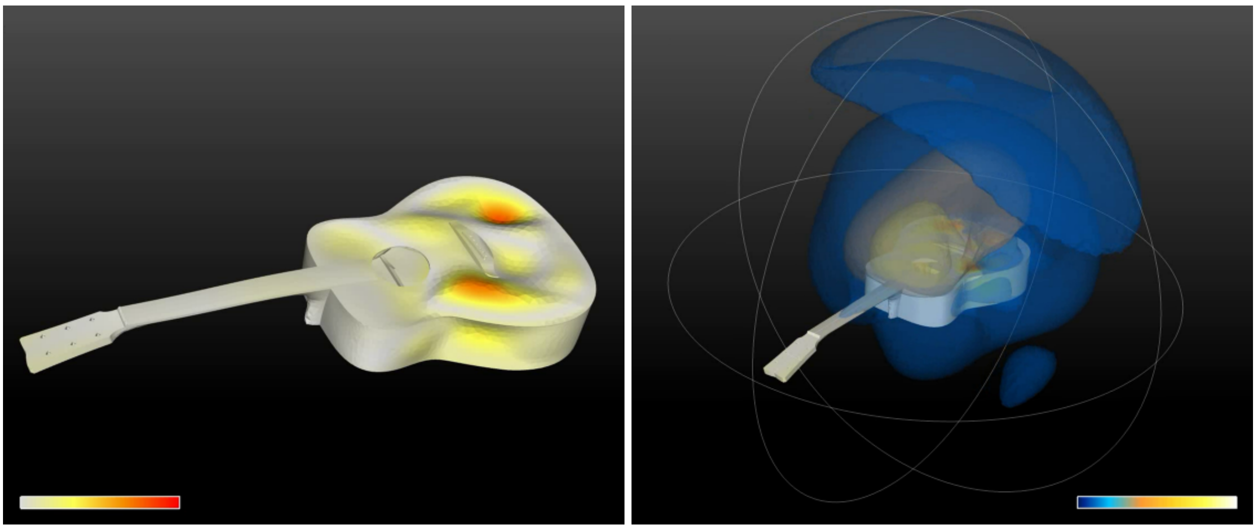

Figure 9: Snapshots of the guitar body displacements (left) and isosurfaces of absolute sound pressure (right) during the time-domain simulation using the ROM.

4 Conclusions and future work

This work demonstrates the benefits of using stability-preserving model order reduction techniques in conjunction with the finite element method for modeling the behavior of acoustic guitars. The resulting reduced-order models are accurate approximations of the finite element model for both frequency-domain and time-domain analysis while significantly reducing the algebraic size of the system and the associated computational cost. The numerical model of a YAMAHA acoustic guitar is shown to closely match the behavior of a real guitar through experimental validation. The development of stable reduced-order models for guitar analysis allows designers and engineers to study the effect of design modifications on the transient deformations and generated sound field of the guitar, even auralization of the sound of the virtual guitar design is possible.

Possible future work could consist in improving the proposed modeling approach as well as exploring new applications. On the modeling side, the use of infinite elements would better approximate the free radiation condition at the boundary, while still allowing for stable model order reduction. A new application could be the use of parametric model order reduction to optimize the design of the guitar or to identify material properties (as is done in [11]).

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References

[1] M.J. Elejabarrieta, A. Ezcurra, C. Santamaria, *Vibrational behaviour of the guitar soundboard analysed by the Finite Element Method*, Acta Acustica united with Acustica, Vol. 87 (2001), pp. 128-136.

[2] M.J. Elejabarrieta, A. Ezcurra, C. Santamaria, *Coupled modes of the resonance box of the guitar*, Journal of the Acoustical Society of America, Vol. 111, No. 5 (2002), pp. 2283-2292.

[3] P. Benner, V. Mehrmann, D.C. Sorensen, *Dimension Reduction of Large-Scale Systems*, Springer-Verlag, Berlin Heidelberg (2005).

[4] E.J. Grimme, *Krylov projection methods for model reduction*, PhD Thesis, ECE Department, University of Illinois (1997).

[5] Z. Bai, Y. Su, *Dimension reduction of large-scale second-order dynamical systems via a second-order Arnoldi method*, SIAM J. Scientific Computing, Vol. 26 (2005), pp. 1692-1709.

[6] A. van de Walle, F. Naets, E. Deckers, W. Desmet, *Stability-preserving model order reduction for time-domain simulation of vibro-acoustic FE models*, International Journal for Numerical Methods in Engineering, doi:10.1002/nme.5323 (2016).

[7] D. Givoli, *High-order local non-reflecting boundary conditions: a review*, Wave Motion, Vol. 39, No. 4 (2004), pp. 319-326.

[8] S. van Ophem, O. Atak, E. Deckers, W. Desmet, *Application of a time-stable model order model reduction scheme to an exterior vibro-acoustic finite element model*, in A. van de Walle, P. Sas, D. Moens, editors, *Proceedings of ISMA2016, Leuven, Belgium, 2016 September 19-21*, Leuven (2016).

[9] G. Flagg, C. Beattie, S. Gugercin, *Convergence of the Iterative Rational Krylov Algorithm*, System & Control Letters, Vol. 61, No. 6 (2012), pp. 688-691.

[10] J. Chung, G. Hulbert, *A Time Integration Algorithm for Structural Dynamics With Improved Numerical Dissipation: The Generalized-alpha Method*, Journal of Applied Mechanics, Vol. 60 (1993), pp. 371-375.

[11] A. van de Walle, L. Rouleau, E. Deckers, W. Desmet, *Parametric model-order reduction for viscoelastic finite element models: an application to material parameter identification*, in *Proceedings of ICSV22, Florence, Italy, 2015 July 12-16*, Florence (2015).